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Application of Fractional Calculus in Analog Signal Processing Circuits

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Abstract: In this Article various methods for realization of fractional element has been presented. This Article brings lot of facts and information for those researchers who are going to begin their research work in the field of fractional calculus and fractional circuits. A lot of motivation and direction for beginners has been given in this article. Also the application and advantages of fractional order over integer order systems for control system, signal processing and biomedicine has been discussed. This Article will validate the advantages of mathematics and will shows the relationship of mathematics with circuits designing. This work will also attract our researchers toward mathematics. The application of fractional element is verified using MATLAB software.

Keywords: Fractional Calculus, FO low pass filter, Continued fraction expansion, rational approximation.

I. INTRODUCTION

An article was published [1] in 2008, introduced fractional order continuous time system as the "21th century systems." Another article published [2] in 2010 validate the benefit of this research area. The invention of the fractional order circuit element has opened the door to various applications with excellent performance, which were not achievable for standard integer order circuit elements. In order to compare with a standard circuit element, a fractional element is a function of parameter value (L or C) and the fractional order α. In general, it is an area where biochemistry, medicine and electrical engineering overlap, giving rise to numerous new potential applications[3].The fractional circuits element and fraction calculus are used in various fields like control system, integrated circuit, electrical, network, transmission line, wave propagation, image processing, medical science and fluid mechanics[4-7]. Recent studies brought Fractional calculus into attention due to the fact that many physical phenomenon in nature exhibits fractional behaviour and cannot be modelled by integer order differential equation. The importance of fractional order models is that they yield a more accurate description and give a deeper insight into the physical process underlying long range memory behaviour [8]. This article organized mainly in four sections, the introduction to calculus and various approximation methods have been discussed in section 2. In section 3, a rational approximation using continued fraction expansion formula has been presented also the application of fractional operator in circuits designing has been presented, and in last simulated result and discussion on the topic has been presented in section 4.

II. FRACTIONAL CALCULUS AND METHOD OF APPROXIMATION

2.1 Background:

Isaac Newton (1642–1727) and Gottfried Wihelm Leibniz (1646–1716) separately founded calculus in the 17th century. In his discovery of calculus, Leibniz first introduced the idea of a symbolic method and used the symbol $\frac{d^n y}{dx^n}=D^n y$ for the nth derivative, where n is a non-negative integer. In a letter to L' Hospital in 1695 Leibniz

raised the following question. "Can the meaning of derivatives with non-integer order be generalized to derivatives with non-integer orders?" L'Hospital was somewhat curious about that question and replied by another question to Leibniz:

"What if the order will be ½?" Leibniz in a letter dated September 30, 1695-the exact birthday of the fractional calculus!- replied: "It will lead to a paradox from which one day useful consequences will be drawn." In these words the fractional calculus was born.

2.2 Methods of approximation:

There are various type of approximation methods available for continuous and discrete time implementation of fractional order element as given below

A. Continuous time approximation methods:
a) Continued fraction expansion
b) Carson's method
c) Matsuda's method
d) Oustaloup recursive approximation
e) Modified oustaloup approximation

B. Discrete time approximation methods:
a) Zoh-zero order hold
b) Foh-linear interpolation
c) Tustin-bilinear approximation
d) Prewarp-tustin approximation with frequency prewarping

Out of these approximation methods for fractional order element, continued fraction expansion formula to be used to realised fractional order device and circuits.

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III. CONTINUED FRACTION EXPANSION FORMULA AND REALIZATION OF FRACTIONAL CIRCUITS

The continued fraction expansion (CFE) formula is used for the rational approximation of fractional operator or the transfer function of fractional device.

The CFE formula is given as [9].

$$(1+x)^{\alpha} = \frac{1}{1-}\,\frac{\alpha x}{1+}\,\frac{(1+\alpha)x}{2+}\,\frac{(1-\alpha)x}{3+}\,\frac{(2+\alpha)x}{2+}\,\frac{(2-\alpha)x}{5+----------} \qquad (1)$$

The above series converges in the finite complex s-plane, along the negative real axis from x= -∞ to x = -1. Now in order to get the rational approximation for $\sqrt{s}$, putting x = s – 1 and taking number of term. For example by taking two numbers of terms, we obtained the first order rational approximation of $\sqrt{s}$ as

$$s^{0.5} = \frac{3s+1}{s+3} \qquad (2)$$

On the same way, by taking more numbers, higher order rational approximation of $\sqrt{s}$ will be given as

Table 1 Rational approximation of s $^{0.5}$

SN	No. of terms	Rational Approximation
1	2	$\frac{3s+1}{s+3}$
2	4	$\frac{5s^2+10s+1}{s^2+10s+5}$
3	6	$\frac{7s^3+35s^2+21s+1}{s^3+21s^2+35s+7}$
4	8	$\frac{9s^4+84s^3+126s^2+36s+1}{s^4+36s^3+126s^2+84s+9}$
5	10	$\frac{11s^5+165s^4+462s^3+126s^2+330s+1}{s^5+55s^4+330s^3+462s^2+165s+11}$

The next step of this work is based on the statement " If mathematics never goes wrong then the above rational transfer function should gives the characteristics of s $^{0.5}$" and also based on newton' s statement "Nature follows mathematics". In this work an effort has been made to incorporate mathematics and circuits designing. The transfer function of equation 2 should give the characteristics of fractional inductor. Further to get the fractional order capacitive nature the numerator and denominator of equation 2 should be interchanged. In order to verify proposed fractional operator, consider a simple low pass filter circuit with a fractional operator as follows.

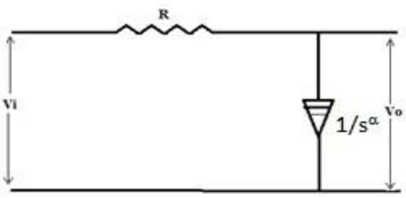

Figure 1 Fractional order low pass filter (FOLPF)

The magnitude and phase of the above transfer function is given by

$$\text{Magnitude} = \frac{\frac{1}{R}}{\sqrt{\omega + \frac{\sqrt{2}}{R}\omega^{\frac{1}{2}} + \frac{1}{R^2}}}$$

$$Phase = -tan^{-1}\left(\frac{\omega^{1/2}}{\omega^{1/2} + \frac{\sqrt{2}}{R}}\right)$$

IV. RESULTS AND DISCUSSON

The fractional order operator of order, -α has been used in the FO low pass filter circuit as shown in the figure 1. The magnitude and phase response of the FO Low pass filter is shown below in figure 2.

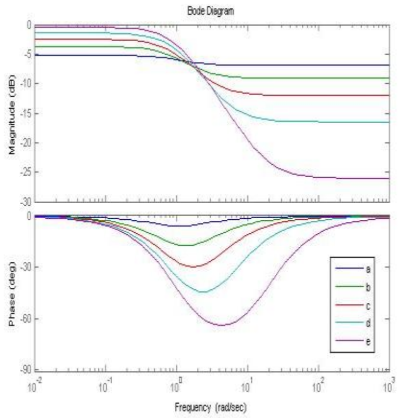

Figure 2 Response of FOLPF for different order (-α), a=0.1, b=0.3, c=0.5, d=0.7, e=0.9

From the result obtained it can be observe that the magnitude and phase move to flattened as the order of the fractional operator decreases. The result in the figure4.1 represents the magnitude and phase of FO low pass filter using $1/_{s^{\alpha}}$ instead of conventional capacitor, where α = 0.1, 0.3, 0.5, 0.7, 0.9 are used in the circuit.

V. CONCLUSION

A Motivation towards designing of fractional order circuits has been given in this article. Different method of rational approximation of fractional operator and further the use fractional operator in circuits designing has been presented in this paper. The output of fractional order low pass filter for different values of α has been simulated by MATLAB software.

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